



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

## HAGEN'S SYNOPSIS OF HIGHER MATHEMATICS.\*

By PROF. WILLIAM BENJAMIN SMITH, Columbia, Mo.

One very grave obstacle to the successful prosecution in the United States of original research in mathematics has been the practical inaccessibility of its literature. Ignorance of this literature, which is immense, has meant ignorance of the results of previous and contemporaneous investigation the world over, as well as of the new outstanding problems disclosed from time to time, no less than of the directions in which energy might be hopefully expended. But while the vast bulk of this literature lay strewn without order in two-score sets of periodicals, and for the most part in foreign languages, it remained a hopeless task for the great body of our youth bent on mathematical study even to use it with much advantage, and far more hopeless to master it. Excellent service to the cause of learning has of late years been rendered in England, and still more upon the Continent, in collecting the widely-scattered memoirs of the great masters and publishing complete editions of their works. This indeed is much, but it is yet far from being enough. A book is worth nothing that is not worth an index, and if in the whole circuit of intellectual achievement there be any results that deserve careful collation and orderly arrangement, it must be in the Higher Mathematics. Even the sight of but one wing or transept in the temple of mathematical knowledge, as of Modern Geometry or the Invariantive Analysis, is awe-inspiring, but how much more a view, as from some higher space, of the entire edifice, not merely as one huge mass but in all the details of its finely articulated structure. Now it is precisely such a view as this, minute and yet comprehensive, that Prof. Hagen has attempted to present in his monumental *Synopsis der Höheren Mathematik*. In four stately volumes he seeks to summarize the results of as many centuries of investigation, and enable the student and explorer to ascertain, without tedious consultations of dispersed and often unobtainable memoirs, the state of knowledge at any given point, to orient himself anywhere, to demark sharply the known from the unknown, and to note the trend and promise of the lines of advancing discovery. A more useful labor than this in the present condition of mathematical literature can hardly be imagined; moreover, it calls for all but the very highest, that is, creative mathematical power; in particular, for

---

\* SYNOPSIS DER HÖHEREN MATHEMATIK. Von Johann G. Hagen, S. J., Director der Sternwarte des Georgetown College, Washington, D. C. Erster Band: Arithmetische und Algebraische Analyse. Berlin, Verlag von Felix L. Dames.

immense erudition, an unerring logical instinct for the often extremely subtle relations obtaining among propositions; but above all, for untiring industry. Once accomplished, however, such a work would be of permanent value, and would lay all future generations, no less than the present, under heavy obligations. It was with lively pleasure, then, that we read the prospectus of Prof. Hagen's undertaking, and the first volume, now before us, meets fully our anticipations. It is an imposing quarto of four hundred pages bearing the imprint of Felix L. Dames, Berlin, and is concerned exclusively with *Arithmetische und Algebraische Analyse*. It falls into twelve *Abschnitte*, each devoted to a *Theorie*, as of *Numbers*, *Complex Magnitudes*, *Combinations*, *Series*, *Product-series and Faculties*, *Continued Fractions*, *Sums and Differences*, *Functions*, *Determinants*, *Invariants*, *Substitution-groups*, and lastly, *Equations*. The first, *Theorie der Zahlen*, is embraced in forty-two pages. When we reflect that Gauss regarded the Theory of Numbers as the very core of Mathematics, in fact almost as Mathematics itself, and all else bearing the name as auxiliary thereto, or at best an application thereof, we are at first surprised to find only one-tenth of one volume out of four dedicated to this section. However, the second section, of twelve pages, carries us into the modern doctrines of Complex Numbers, Gaussian, Kummerian, ideal, and universal, and so in a measure restores a juster proportion. Moreover, not a little that one person might range under the Theory of Numbers another might assign to the Theory of Equations or to Combinatorial Analysis; and we do, in fact, find many properties of numbers in these two sections. Only a very minute inspection could show, then, that this leading division of the work had been slighted. Besides all this, the profounder theories of Kummer and Kronecker, of Dedekind and Weierstrass, have been only broached as yet, and are very far from complete development.

Again, we find forty-two pages devoted to the *Theory of Functions*. At once we think of the Theta-functions of Jacobi, and of the Weierstrassian *Sigma* and *P*<sub>*e*</sub> functions, creations that of themselves have widely extended the range of mathematical power, and we wonder whether the author has succeeded in tabulating in forty-two pages such a vast body of doctrine. But the title-page reminds us that this volume treats only of Arithmetical and Algebraic Analysis, and accordingly only so much of *Function-Theory* is here summarized as belongs properly to Algebra. For the great creations of the last two generations in Elliptic and Hyperelliptic Functions we must wait on the fourth volume. Hence it appears that any criticism upon any apparent deficiency or inadequacy must run great risk of turning out to be premature. The principles of division adopted in the text may or may not recommend themselves to the reader; but in such matters there is ample room for diversity of judgment.

Nowhere else in the whole range of human intellectual endeavor are doctrines the most various so interpenetrative as in Mathematics. Each discipline is indeed, so to say, a personality, existing in and for itself, yet each is inextricably interlaced with every other. The realm of Mathematics is thus a conform depiction of the universe itself, a veritable chart of Nostradamus,

Wo Alles sich zum Ganzen webt,  
Eins in dem Andern wirkt und lebt.

It is not strange, then, that several classifications may have equal logical justification, and what we do not find where we first seek it may with good reason be introduced elsewhere. We have not read all of these four hundred quarto pages, but we have consulted them repeatedly on almost every variety of subject they contain, and though occasionally disappointed at first, never without reasonable satisfaction in the end. The work abounds in cross references that are very convenient, while the references to original authorities are exceedingly helpful. *Scire ubi aliquid inveniendum, magna pars scientiæ.* The historical data interspersed throughout the book appear to have great value. One always feels much nearer to a theorem, knowing its date, its discoverer, and some of its antecedents. We also thank the author for his *Vorbemerkungen*, which are in general very judicious.

Examining the work we seem to be looking at some precious Florentine mosaic, where the *disjecta membra* of four centuries have been so patiently collected and skillfully composed as to present the appearance of a single organic whole. In reality the work has been the slow and toilsome deposition of a quarter-century. It is impossible, then, that all parts should show equal exhaustiveness, if indeed equal carefulness. Some of the more ancient results are accessible in a hundred well-known volumes; some of the more modern perhaps in only a single rare number of an obscure periodical. It is not surprising, then, if the *Theory of Invariants* appear less fully treated than such a subject as the *Theory of Equations*. But on the whole, the completeness of the exhibit is admirable; and the immense agglomerate of doctrine has been fused into a well-nigh homogeneous mass. Throwing open the book at various select critical points we find a number of pencil marks made on first reading, where the statements, if not incorrect, seem at least inadequate, or unguarded, or perhaps pronounced from a standpoint already overcome. But inaccuracies lie concealed in the most conscientious writings, and it would be a fine sieve that should strain out every one in transference to such a *Synopsis*. Some of these have already caught the author's eye and are noted among the *Berichtigungen*, along with a few errors of type. Of these latter also, however, some remain uncorrected; as on page 7, line 2, where  $a$  and  $b$  should be  $\alpha$  and  $\beta$ . But the general mechanical execution is excellent.

This is preëminently an age of great mathematicians. Not even the younger sciences, as Biology, are advancing more rapidly than their eldest sister. If the radius of the sphere of mathematical lore be not augmenting faster than ever, at least the volume certainly is. Three more tomes, companions of this under review, are promised. Before even diligence can hurry them through the press, great additions will surely have been made to our present treasures, and a large supplement will be needed. Elegantly dedicating his high emprise *Almæ Georgiopolitanæ Academiæ*, the author piously invokes Divine aid to her noble work of culture, as she auspiciously opens her second *sæculum*; nor perhaps would a similar sentiment unfittingly close this inadequate notice. Assuredly every lover of learning must pray that Prof. Hagen may carry forward his work to completion even beyond the wide confines of its original conception. But however this may be, what has already been accomplished is of great and lasting value, and establishes a secure claim to the gratitude of all students of Mathematics.

UNIVERSITY OF MISSOURI, *September* 19, 1892.